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REFINED UNCERTAINTY BUDGET FOR REFERENCE TORQUE CALIBRATION FACILITIES

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Abstract – Contributions of uncertainty budgets suitable for special applications of reference torque calibration facilities are calculated with a Monte Carlo method implemented in a spreadsheet software application. The simulations were combined with measurements of parameters with a high number of observations and compared with usual simplified uncertainty estimations for examples of uncertainty contributions concerning traceability of voltage ratio, noise, resolution and position effect. The results are more differentiated and detailed than usual GUM estimations and provide deeper understanding about uncertainty contributions than with GUM methods exclusively. New proposals for the uncertainty estimation of the examined contributions are offered.

***Keywords*: uncertainty, GUM, Monte Carlo, torque, reference**

1 INTRODUCTION

When the PTB-2-kN·m reference torque calibration facility was put into operation more than 15 years ago, it was the first of its kind to be in service in a national metrology laboratory. Since then its best measurement capability (bmc) of $2 \cdot 10^{-4}$ has been confirmed in several comparison measurements. Nevertheless, the uncertainty budget established when reference torque calibrations first began no longer meets all the requirements of today's applications. Not only have a lot of investigations on specific uncertainty contributions provided more and detailed knowledge about the potentials and limitations of reference calibration procedures, but also the requirements on these procedures had developed into some divergent branches. Thus on the one hand key comparisons with highest efforts to achieve the best possible measurement uncertainty are within the scope of reference torque calibration facilities at the Physikalisch-Technische Bundesanstalt (PTB) and on the other hand, there are application-oriented measurements with special features like continuous load, releasing operation, partial sequences, low signal saturation or prototype sensing chains. Therefore, more differentiated and detailed uncertainty budgets are requested, which should give the opportunity to represent each of these particular processes in a specific manner. Here, the uncertainty simulation – in particular the Monte Carlo method - can help to investigate questions, which are not easily accessible by measurement.

Additionally, uncertainty budgets of calibration laboratories operating reference torque calibration facilities are to be confirmed during accreditation processes. Here also a deeper understanding of uncertainty calculations for reference torque calibration is useful.

2 METHODS

Usually, uncertainty budgets are performed under certain assumptions that allow tight and manageable practices. One convenient way is, to use uncertainty calculation instructions of an existent guideline as a template. Especially laboratories calibrating in the field of transfer torque transducers according to DIN 51309 [1] might be tempted to employ the methods of this guideline for the uncertainty budget of their calibration facility. In order to assess how far these simplified uncertainty estimation (SUE) methods are useful in this context, examples of uncertainty contributions are compared with enhanced uncertainty estimations (EUE), which follow the Guide to the Expression of Uncertainty in Measurement (GUM) [2] much more strictly, and with uncertainty calculations using the Monte Carlo method (MCM) [3]. In this comparison, the result of the MCM represents the smallest possible uncertainty estimation of the studied conditions.

With the help of the MCM, the adequate confidence interval can be calculated by the numerical integration of the relevant probability density function (PDF) (Fig. 1). The integration has to optimize the expectation value ε, which is the middle of the confidence interval, to a value which allows the smallest possible confidence interval. Because the MCM also provides the empirical standard deviation σ, the coverage factor k can be found.

Often, a combination of the MCM with empirical data using a great number of observations n provides a more detailed evaluation of the uncertainty contributions concerned. The MCM is implemented in a spreadsheet software application in order to obtain both free control for manipulating the synthetic data and free access to the information about the stochastic results.

The synthetic input data for the MCM can be generated according to the results of measurements with a high number of observations. Hence, the standard uncertainty σ, the expectation value ε and the probability density function (PDF) can be adjusted according to real conditions. The results of the simulation are thus significant for reality in return.

Often it is useful to truncate the synthetic deviations to an amount of ± 3 σ. Wherever higher deviations would be discarded as outliers in real measurement, this method is reasonable.

3 RESULTS

The MCM can be used to study in detail a great variety of uncertainty related questions in the field of reference torque calibration. Some of the benefits of the MCM could not be achieved performing real measurement or GUM estimations exclusively.

3.1 Probability distributions

The GUM offers routes for the calculation of the influence of parameters as a type B evaluation, when a statistical analysis is not accessible. Often the choice of an adequate probability distribution is dominated more by principle considerations than by exact knowledge.

Here the MCM can give insights into the behaviour of parameters even if they are correlated with others. Provided that an adequate model function of the effect can be set up, numerical simulations with $n \approx 10^5$ can give a clear view of the probability distribution.

One example is the influence that finite amplifier resolution exerts on the uncertainty of measurements. This contribution is normally distributed with usual assumptions. In MCM simulations this can be confirmed and the standard deviation is found to be in the range of a noise level as expected; just as for the coverage factor of about 2 (Fig. 1).

An MCM scan (Fig. 10) however unveils that under certain conditions the distribution function changes into a kind of beta function with a coverage factor of 1.5 and a standard deviation six times higher than normal (Fig. 2). This behaviour can be taken into account by effectual estimations as shown later in this paper. Or, if stricter uncertainty budgets are needed, a direct calculation with the MCM with measured input data is possible.

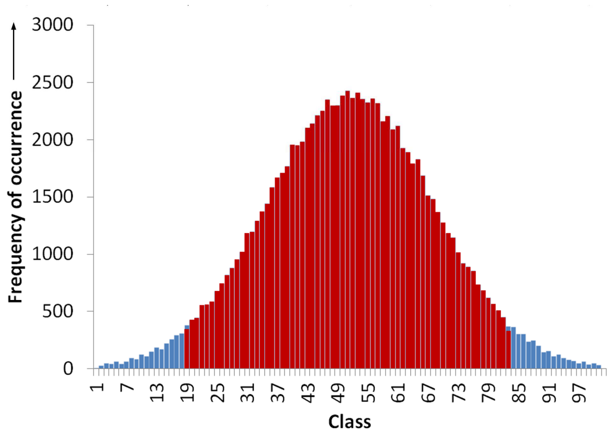

Fig. 1: Probability distribution (blue) of the uncertainty contribution of a resolution of $8 \cdot 10^{-6}$ mV/V at a signal of 1.3 mV/V with noise of $5 \cdot 10^{-7}$ mV/V. The red area covers the minimum confidence interval of 0.95. The coverage factor is 1.97, the standard deviation is $4.94 \cdot 10^{-7}$.

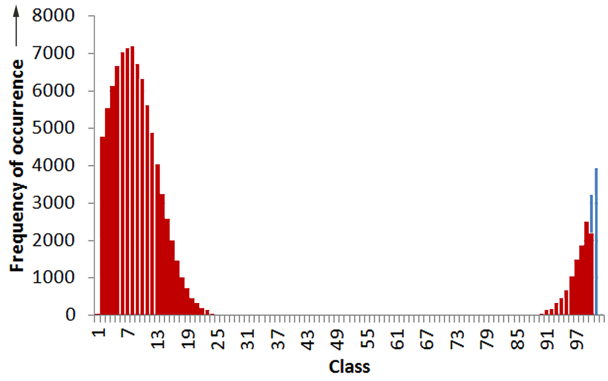

Fig. 2: Probability distribution of the uncertainty contribution of a resolution of $9 \cdot 10^{-6}$ mV/V at a signal of 1.3 mV/V with noise of $5 \cdot 10^{-7}$ mV/V (blue). The red area covers the minimum confidence interval of 0.95. The coverage factor is 1.50, the standard deviation is $2.98 \cdot 10^{-6}$.

3.2 Uncertainty of data fit

The approximation of hypothetic functions to measurement data is a capable method of achieving coefficients for a certain effect. However, it is an extensive task to calculate the standard uncertainties of these coefficients. In the GUM, an analytical method is given for approximations with linear functions. But there are some reasons to depart from this method sometimes.

The GUM recommendation only measures the effect of noisy expectations of the input data along an altered quantity – in other words: it only deals with the deviation of the data points from the linear approximation, but it ignores the influence of the variance of each of the data points.

Moreover, for approximations with more complex functions than linear ones, there is no easy analytical access.

Furthermore, the GUM calculation generally deals with differences between the fit and the measured data as a deviation to the fit. In practice, however, a fitted function might reflect a physical effect better than the corresponding measurement data, and therefore the measured data would bear the deviation. In this case the approximation produces a benefit to the calculated coefficient because of its averaging effect on the noisy data.

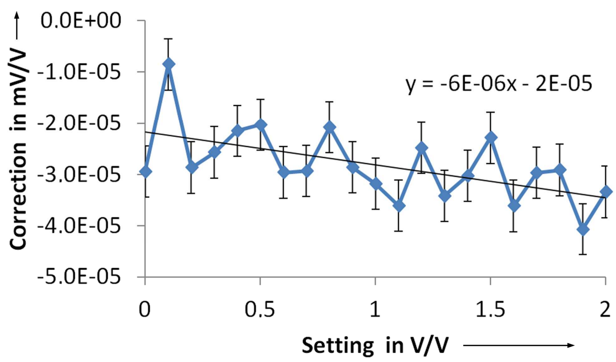

Fig. 3: Determination of a linear approximation of the correction value for a BN calibrated at the national standard of voltage ratio. The data points of the calibration are modified by the MCM. One of 10^5 simulated approximations is shown.

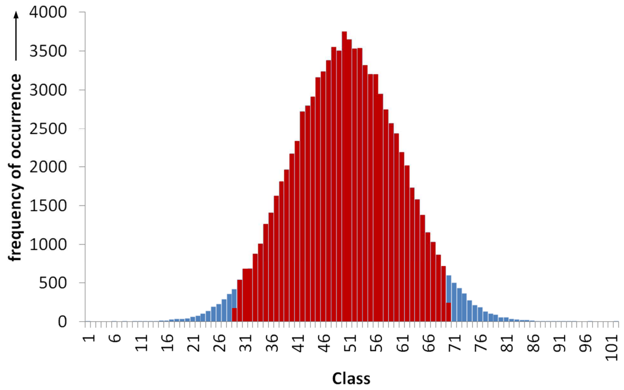

Fig. 4: Probability density function (blue) of the slope found in the linear approximation of the traceability calibration on a bridge normal using synthetic 3-σ-truncated deviations of the calibration values with normal distribution. The red area covers the minimum interval of 0.95 confidence under variation of the expectation value.

An example is the traceability calibration of a bridge standard (BN) which is used in the traceability chain of voltage ratio measurements. The relative uncertainty w_{BN} of each calibration value provided by the national standard for voltage ratio is dominated by the relative resolution $w_D = 1 \cdot 10^{-7}$ of the applied voltage divisors with the dividing ratios $D_1 = 0.05$ and $D_2 = 0.04$ [4, 5]:

$$w_{BN} \approx w_D \sqrt{2(D_1^2 + D_2^2)} \quad , \qquad (1)$$

which leads, for the applied measuring range of 2 mV/V, to a constant uncertainty contribution of $u_{BN} = 5 \cdot 10^{-6}$ mV/V over the range from 0.1 mV/V to 2 mV/V.

A numerical simulation of the national voltage ratio standard with normal distributed noise of $\sigma = 5 \cdot 10^{-6}$ mV/V truncated at 3 σ for all of the 21 calibration points leads to a more or less different slope in each of the arbitrary combinations. In Fig. 3, this situation is illustrated for one of the 10^5 calculated combinations of calibration points. At the end of the MCM process, the slopes are distributed as shown in Fig. 4 and the numerical integration yields the expanded uncertainty of the slope of $U_{BN} = 3.5 \cdot 10^{-6}$ mV/V.

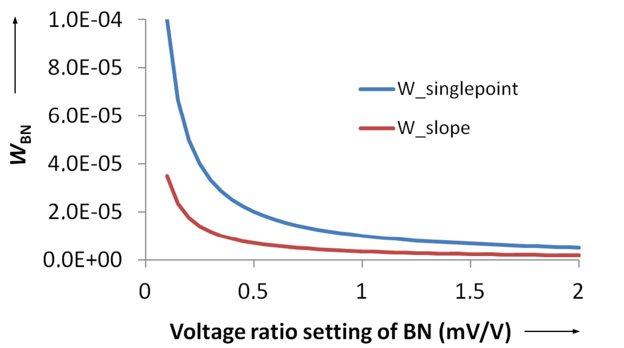

Fig. 5: Relative expanded uncertainty contribution of the traceability calibration of a bridge standard using the uncertainty of each single-point value (blue) and using the uncertainty of the linear approximation found by the MCM (red).

Using this value, the relative contribution of the BN traceability is less than $4 \cdot 10^{-5}$ in the range between (0.1 ... 2) mV/V (Fig. 5), which is tolerable for the aim of a total expanded uncertainty of $W_{KE} = 2 \cdot 10^{-4}$ of the calibration facility. If smaller voltage ratios than 0.1 mV/V are needed, differentiated measurement uncertainty budgets can be stated.

As depicted in Fig. 5, the estimation of the uncertainty contribution of the BN calibration on the basis of each single calibration point would result in much higher uncertainty restrictions of the calibration facility.

3.3 Systematic deviations

Though in the GUM avoiding the following is recommended, the consideration of systematic deviations as uncertainty contributions is widely used when the amount of the effect seems to be small compared to the combined uncertainty of the complete system. The strict method is to correct the effect using a coefficient and to take the uncertainty of the coefficient into account, regarding the appropriate sensitivity coefficient.

Reference torque transducers are to be calibrated in the national standard calibration facility before being used in a reference calibration facility. In Germany this has to be performed according to the guideline DIN 51309 [1]. There, the influence of the transducer position is determined as a span b_{Pos} of the three positions 0°-120°-240°:

$$b_{Pos} = \mathrm{Max}(S_{0°}, S_{120°}, S_{240°}) - \mathrm{Min}(S_{0°}, S_{120°}, S_{240°}) \quad . \qquad (2)$$

Often, the uncertainty contribution of the reference transducer position during its use in a calibration facility is estimated by the amount of b_{Pos}. There are however two problems: Firstly, b_{Pos} is a contribution that consists of the transducer sensitivity to disturbing effects like cross forces multiplied by the currently acting cross forces in the national standard facility. But this is not what a reference transducer will experience in the reference calibration facility. Therefore, in the uncertainty budget b_{Pos} has to be replaced by the equivalent parameter achieved by rotations in the reference calibration facility itself.

Secondly, b_{Pos} could not consider the position effect completely. Since the effect bases on a sinusoidal function, the span will leave always a systematic rest in the budget. Since the effect is based on a sinusoidal function, the span instrument cannot provide exact mean values in the case of position changes of 120° and therefore always leaves a systematic rest in the uncertainty budget.

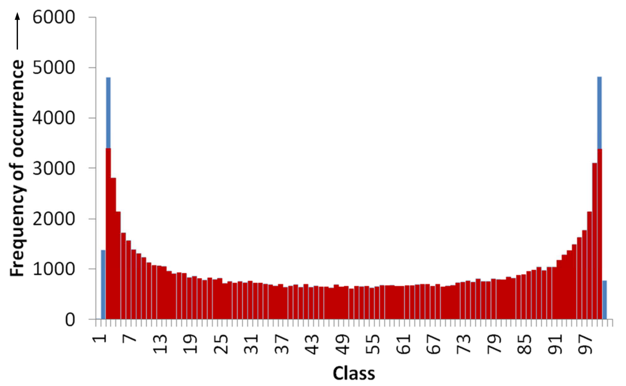

Fig. 6: Probability density function (blue) of the deviation due to the uncertainty of the corrective value $b_{P,corr}$ concerning the position effect of a reference transducer. The red area covers the minimum interval of 0.95 confidence.

The most rigorous method is to define a fixed working position of the reference transducer in the reference calibration facility, and to find a corrective value b_P from the difference between the average sensitivity $\bar{S}_{0°,120°,240°}$ of three positions measured in the calibration facility in question and the sensitivity related to the working position $S_{0°}$ in this facility:

$$S = S_{0°} + b_\mathrm{P} \quad , \quad (3)$$

$$b_\mathrm{P} = \bar{S}_{0°,120°,240°} - S_{0°} \quad . \quad (4)$$

Then, the residuum uncertainty of the correction $u(b_{\mathrm{P,corr}})$ has to be taken into account in the uncertainty budget. The PDF of this contribution proves to be a beta function (Fig. 6) which is getting sharper with decreasing uncertainty of the positioning angle $\delta(\alpha_\mathrm{Pos})$.

In Fig. 7, the simulated impact of the angle uncertainty of the 120°-position alterations during the determination of $u(b_{\mathrm{P,corr}})$ with a position effect amplitude of $A_\mathrm{Pos} = 1 \cdot 10^{-4}$ mV/V is shown. The influence of the angle uncertainty follows a cubic function and can be neglected as long as the repeatability of the angle setup is better than 0.04 rad (2.3 degrees), which is a realistic request, because in the considered facility this threshold would produce an interval of more than 20 mm at the circumference of the axis. In this way, an estimation of the needed precision in positioning the transducer was possible. What is more realistic is a scenario, where the deviation due to the reference position effect remains uncorrected and is treated as an uncertainty contribution. In Fig. 8 the expanded contribution of the uncorrected effect calculated with the MCM, $U(b_\mathrm{MCM})$, is compared with $U(b_\mathrm{Pos})$ as defined in [1], with $U(b_\mathrm{P})$ and with the correction deviation $U(b_{\mathrm{P,corr}})$ as a function of the positioning angle uncertainty $\delta(\alpha_\mathrm{Pos})$. $U(b_\mathrm{MCM})$ here represents the smallest uncertainty to be expected without correction. The contribution of $U(b_\mathrm{Pos})$ turns out to be significantly smaller than $U(b_\mathrm{MCM})$. It provides in this situation an underestimated uncertainty of almost the level of the correction residuum $U(b_{\mathrm{P,corr}})$. In contrast, $U(b_\mathrm{P})$ keeps close to the MCM reference without falling below it. $U(b_\mathrm{P})$ therefore is appropriate to quantify the influence of the position effect.

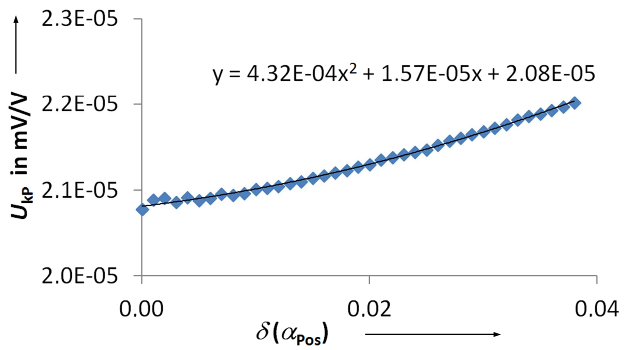

Fig. 7: Simulated standard uncertainty contribution due to the effect of the reference transducer position in the calibration facility as a function of the uncertainty of the 120°-position changes. The results are close to a cubic function given as a black line. The recreation uncertainty of the marked working position of the reference transducer was set to 0.0005 rad and $A_\mathrm{Pos} = 1 \cdot 10^{-4}$ mV/V.

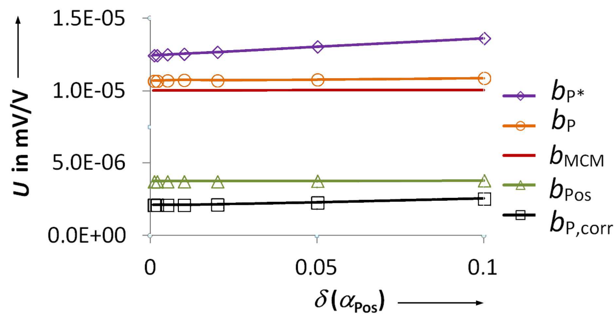

Fig. 8: Extended uncertainty contribution of the position effect obtained with different calculation methods (see text). Parameters: $S_0 = 1.3$ mV/V; $A_\mathrm{Pos} = 1 \cdot 10^{-5}$ mV/V.

While it is a great effort to determine the value of b_P and its standard deviation by repeated measurements in three positions, an analytical approach permits easier uncertainty estimation:

$$U(b_\mathrm{P}^*) = \frac{A_\mathrm{Pos}}{3\sqrt{2}}\{[2+\sqrt{3}][\sin(\delta(\alpha_\mathrm{Pos})) + \cos(\delta(\alpha_\mathrm{Pos}))]\} \quad . \quad (5)$$

Here, only the knowledge of the amplitude and the angle uncertainty is necessary. In practice, with a single sequence of three positions the amplitude can be found by fitting. And the angle uncertainty can be determined by mechanical investigations at the calibration setup, which can be performed much faster than multiple sequences of loadings. In comparison to b_P, this method exhibits a considerably higher estimation (Fig. 8, purple line), but it could be a helpful alternative in case of uncertainty budgets featuring some scope.

The correction of the position effect is a quite extensive method but can reduce the contribution to the uncertainty budget by about a factor 5 (Fig. 8, black line).

3.4 Signal processing

Signals from instruments are influenced by resolution and noise. The resolution can be characterized by the span b_r and cause an equal distribution of values. The noise can be assumed to be normally distributed; its influence can be measured by the standard deviation $\sigma(\bar{S})$ of the mean value of the signal S, when averaged over n observations:

$$\sigma(\bar{S}) = \frac{R_\mathrm{a}}{\sqrt{n}} \quad , \quad (6)$$

being R_a the standard deviation of the signal S.

The expanded contribution to the uncertainty $U_{\mathrm{Ra,br,GUM}}$ of both effects for a GUM conform estimation - with the coverage factor $k = 2$ for a confidence level of 0.95 - is then given by:

$$U_{\mathrm{Ra,br,\,GUM}} = 2\sqrt{\left(\frac{R_\mathrm{a}}{\sqrt{n}}\right)^2 + \left(\frac{b_\mathrm{r}}{2\sqrt{3}}\right)^2} \quad . \quad (7)$$

Here the noise R_a is a theoretical value, which can be indicated only for synthetic signals like in the MCM. In practical applications, the influences of noise and resolution cannot be separated. Therefore, the observed standard deviation of measurements $R_{\mathrm{a,obs}}$ includes the correlation with b_r too. Additionally, real measurements are often limited by a finite number of observations. Both effects can be studied with the MCM and the statistical values of $R_{a,obs}$ as functions of n can be found (Fig. 9).

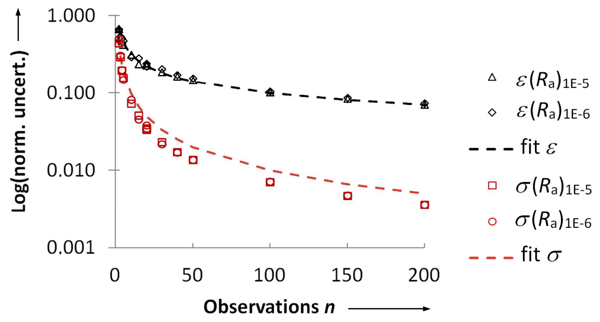

Fig. 9: A simulated determination of the observed noise $R_{a,obs}$ (here shortened to R_a) as a function of the number of observations n gives the expectation value proportional to $n^{-0,5}$ and the standard deviations proportional to n^{-1}. The dashed lines represent fitting functions using these proportionalities. The ordinate is given as a normalized axis. Two sets of data are processed with $R_a = 1\cdot 10^{-5}$ mV/V respectively $R_a = 1\cdot 10^{-6}$ mV/V and $b_r = 1\cdot 10^{-6}$ mV/V.

Furthermore, the MCM demonstrates that $k = 2$ is not given exactly in all combinations of R_a and b_r, and that for noise levels in the range of the resolution, the rectangular distribution function of the combination can change into a beta distribution, depending on the position of the resolution window relative to the expectation value of the signal. This beta function gives rise to unusual behaviour of the uncertainty contribution of resolution as a function of resolution and noise. In normal mode, the uncertainty vanishes rapidly if R_a decreases to values smaller than b_r (Fig. 10, C). In the case of beta distribution, the uncertainty increases in this area for decreasing R_a to values in the range of b_r (Fig. 10, A). For R_a much higher than b_r, the uncertainty contribution is stable at a value that GUM predicts (Fig. 10, B).

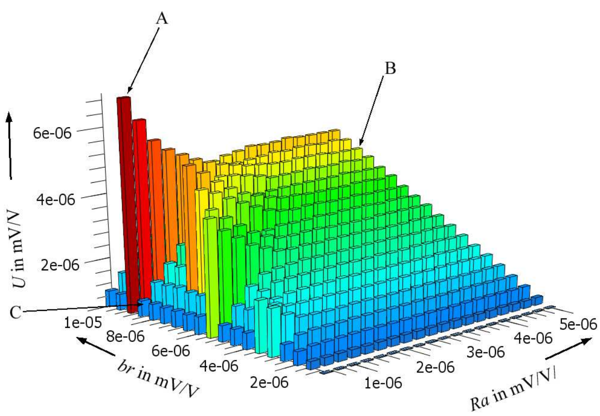

Fig. 10: Simulated expanded uncertainty contribution of resolution at a confidence level of 0.95 as a function of noise and resolution.

The transition between cases A and C in is sharp, so the contribution of b_r could be expressed in a split formula with empirical borders:

$$U_{br} = \begin{cases} 2\sqrt{\left(\frac{b_r}{2\sqrt{3}}\right)^2}, & \frac{0{,}15\cdot b_r + 5\cdot 10^{-7}}{R_a} < 1 \\ 2b_r, & \frac{0{,}15\cdot b_r + 5\cdot 10^{-7}}{R_a} \geq 1 \end{cases}. \qquad (8)$$

To simplify the calculation and especially for low noise measurements, the lower term can serve as an upper estimation of the uncertainty contribution of resolution.

Using the information of the detailed MCM investigations described above, an enhanced version of (7) can be expressed:

$$U_{Ra,br,\,EUE} = 2\sqrt{\left(R_{a,obs}\frac{\sqrt{n+1}}{n}\right)^2 + (b_r)^2} \quad . \qquad (9)$$

In the SUE method, not the standard deviation R_a is used to characterize the influence of noise, but the series range R_b, which is defined as the span of measured maximum and minimum of the signal. With the help of this instrument, an estimation of uncertainty in two cases is expressed:

$$U_{Ra,br,\,SUE} = \begin{cases} \sqrt{\left(\frac{\frac{R_b}{2}+b_r}{2\sqrt{3}}\right)^2 + \sigma^2(R_b)}, & R_b > b_r \\ \sqrt{\left(\frac{b_r}{2\sqrt{3}}\right)^2 + \sigma^2(R_b)}, & R_b \leq b_r \end{cases} \quad . \qquad (10)$$

An MCM analysis shows, that R_b is increasing with n but not converging (Fig. 11), while the standard deviation of R_b is roughly constant at the value of about R_a. As MCM simulations show, the expectation value of R_b can be calculated approximately by

$$\varepsilon(R_b) \approx (1 + 0.9\,\ln(n))\cdot R_a \quad . \qquad (11)$$

As shown in (Fig. 11), $\varepsilon(R_b)$ quickly increases with n to values five times higher than R_a. Therefore, the second case of (10) will only be valid for measurements with very poor resolution or extremely low noise or both.

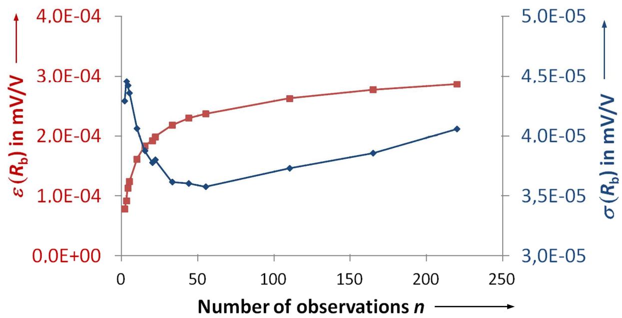

Fig. 11: MCM analysis of a signal with an expectation value of 1.3 mV/V, and with $R_a = 5\cdot 10^{-5}$ mV/V and $b_r = 1\cdot 10^{-6}$ mV/V. Expectation value $\varepsilon(R_b)$ and standard uncertainty $\sigma(R_b)$ of the series range R_b are shown in dependency of the number of observations n.

A comparison of the MCM, the GUM, EUE and SUE methods for a typical situation in torque calibration with $b_r = 1\cdot 10^{-6}$ mV/V and noise between $1\cdot 10^{-6}$ mV/V and $1\cdot 10^{-5}$ mV/V using (7), (9) and (10) is shown in Fig. 12.

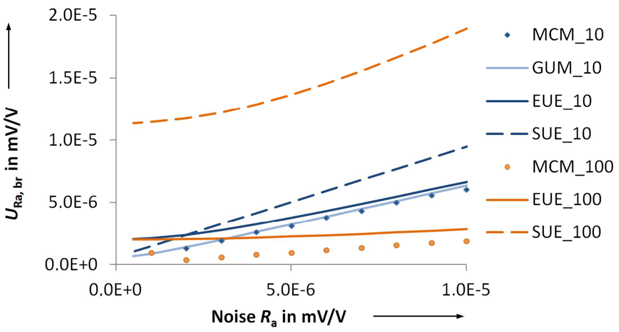

Fig. 12: Combined standard uncertainty contribution $U(R_a, b_r)$ of noise R_a and resolution b_r to a signal with an expectation value of ε = 1.3 mV/V. The results of an MCM analysis are compared with estimations according to the GUM, EUE and SUE methods obtained from 100 or respectively 10 observations.

For higher numbers of observations, the SUE calculation is highly overestimated, which corresponds to the intention of this tool, which is meant to be used for type B uncertainty estimations. But at small n, the decrease of U with R_a is too fast, so for low noise measurements, SUEs could be underestimated.

The GUM calculation is very close to the MCM simulation, but this only works, if a precise value of R_a is available. If this is not provided and in the case of low noise measurements, this calculation is in danger of underestimation, especially if the contribution of b_r becomes beta distributed as discussed above.

The EUE calculation avoids these problems, provides a slight overestimation at uncritical situations with high values of n and R_a, but keeps sufficient distance to the MCM where it is necessary: in the field of high precision measurements. Therefore, EUEs are to be preferred for uncertainty estimations of high grade comparison torque calibration facilities like those at the PTB.

3.5 Coverage factor

The calculation of a combined expanded uncertainty is typically performed with a coverage factor of 2 according to a confidence level of about 0.95. This is only permitted if the Central Limit Theorem (CLT) is valid. To decide this question, evidence about the variance of the combined effects of a measurement is necessary. Usually, in SUE budgets, an estimation is made about the proportion of normally distributed values to those with deviant distributions. If the result is, that the dominant contributions are of the normal distribution, then coverage factor 2 is regarded as to be appropriate. An exact verification of the validity of CLT is practically only possible with the MCM.

Also reliant on the validity of CLT is the calculation of the effective degree of freedom (EDF) according to Walch-Satterthwaite [2], which provides an altered coverage factor k_{eff} considering the reliability of the standard uncertainties due to the number of observations.

As mentioned above, complex uncertainty budgets could be simulated completely with the MCM in order to obtain realistic statistical values without the need to fulfil assumptions like CLT.

A simplified calculation of the combined uncertainty including the contributions discussed in this paper which is inspired by the DIN 51309 is possible with

$$U_{\text{comb, SUE}} = k_{\text{eff}} \sqrt{u_{\text{BN}}^2 + 2\left(\frac{R_b + b_r}{2\sqrt{3}}\right)^2 + \left(\frac{b_{\text{Pos}}}{2\sqrt{3}}\right)^2} \quad , \qquad (12)$$

where b_{Pos} is the observable contribution of the span induced by the position effect discussed in 3.3 which is affected by resolution and noise effects.

Taking into account the conclusions of the MCM investigations described in this paper, an enhanced combined uncertainty budget can be proposed

$$U_{\text{comb, EUE}} = k_{\text{eff}} \sqrt{u_{\text{BN}}^2 + 2\left(R_a \frac{\sqrt{n_{\text{Ra}} + 1}}{n_{\text{Ra}}}\right)^2 + b_r^2 + \left(\frac{b_P}{\sqrt{2}}\right)^2} \quad , \qquad (13)$$

with b_P being the observable deviation due to the position effect according to (4) including influences of resolution and noise as well and being distributed according to a beta function.

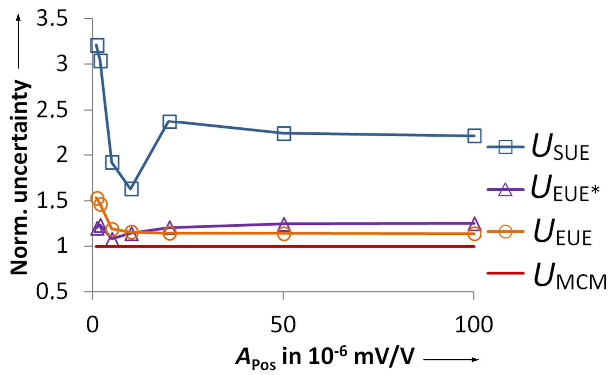

Fig. 13: Normalized expanded combined uncertainty of the contributions concerning traceability of voltage ratio, noise, resolution and position effect according to different uncertainty concepts as a function of the position effect amplitude normalized to the MCM result.

In Fig. 13, these two concepts of uncertainty budgets are compared with the result of the MCM as a function of the position effect amplitude A_{Pos} using the typical parameters given in Table 1.

Tab. 1: Conditions of uncertainty budget comparison shown in Fig. 13. In the lower part results for the working points of three values of A_{Pos} are given.

Parameter	Value		
S_0 in mV/V	1.3		
u_{BN} in mV/V	$1.8\cdot10^{-6}$		
R_a in mV/V	$2\cdot10^{-6}$		
b_r in mV/V	$1\cdot10^{-6}$		
$n_{Ra,r}$	100		
n_{Pos}	3		
n_{Pos*}	100		
$\delta(\alpha_{Pos})$ in rad	0,02		
A_{Pos} in mV/V	$1\cdot10^{-6}$	$1\cdot10^{-5}$	$1\cdot10^{-4}$
U_{MCM} in mV/V	$3.9\cdot10^{-6}$	$1.2\cdot10^{-5}$	$1.0\cdot10^{-4}$
U_{SUE} in mV/V	$1.3\cdot10^{-5}$	$1.9\cdot10^{-5}$	$2.2\cdot10^{-4}$
U_{EUE} in mV/V	$6.0\cdot10^{-6}$	$1.4\cdot10^{-5}$	$1.1\cdot10^{-4}$
U_{EUE*} in mV/V	$4.6\cdot10^{-6}$	$1.4\cdot10^{-5}$	$1.3\cdot10^{-4}$
$k_{eff, SUE}$	2.45	2.57	4.30
$k_{eff, EUE}$	2.13	2.10	2.14
$k_{eff, EUE*}$	2.01	1.98	2.01

Since the results are normalized to the result of the MCM, the latter is represented by a constant line at the value 1. SUE and EUE results are found to be greater than 1, so these methods provide overestimations as they should. In the case of SUE, also underestimation is possible at different combinations of parameters, as shown in section 3.3. In the case of Fig. 13, the overestimation of noise and resolution compensates the underestimation of the position effect. Under different combinations of conditions this compensation is not assured.

With both methods, the results increase with decreasing A_{Pos}, where the relative influence of resolution gets higher. Altogether, EUE stays much closer to the MCM than SUE. EUE therefore is the method to choose for tight uncertainty budgets as often needed in high level laboratories.

EDF analysis results in effective coverage factors k_{eff} close to 2 for EUE and increasing up to 4.3 for SUE. This demonstrates that in certain situations the assumption of $k = 2$ of the SUE method may be too optimistic. The EUE budget on the other hand appears in the parameter field of comparison torque calibration to be dependably close to the MCM simulation even with EDF extension of the coverage factor.

By replacing b_{P} by $b_{\mathrm{P*}}$ in (13) - corresponding to (5) - an uncertainty $U_{\mathrm{comb,EUE*}}$ with analytical estimation of the position effect can be calculated. The benefits of this estimation are discussed in section 3.3. Additionally, the analytical approach of this method permits the estimation to be declared as type B with a high number of observations in the EDF calculation. In comparison with $U_{\mathrm{comb,EUE}}$, the analytical method leads to results closer to the reference of U_{MCM} at low values of A_{Pos}. Altogether these two methods are equivalent in the examined range of parameters.

4 CONCLUSIONS

Using a combination of the MCM and measurements of high number of observations, the calculation of measurement budgets for reference torque calibration facilities can be performed in more detail and more reliably than with GUM estimations exclusively.

The MCM allows separated investigations of individual uncertainty related effects. A deeper understanding of the effects therefore is possible and helps to evaluate necessary simplifications in the uncertainty budgets. The influence of noise and resolution could be considered in a new uncertainty estimation which covers the specific conditions of high level references torque calibrations better than the usual estimations. Furthermore, using the MCM helps to estimate the contribution of voltage ratio traceability much more tightly than without and permits a differentiated view on the limitations of simplified uncertainty budgets.

The compliance between the MCM and most results of GUM estimations validates the suitability of the MCM procedure developed. Differences between the MCM and existing calculation instructions in guidelines are due to differences in the concept of considering the uncertainty contributions in question.

The estimation of the applicable coverage factor using the EDF method can as an example be checked with the MCM of the complete uncertainty budget. In doing so parameters found by deeper MCM investigations and measurements for each parameter are used. This can help to evaluate the quality of the estimation via EDF, to find uncertainty estimations closer to minimum or to confirm the simplifying assumption of k=2.

5 REFERENCES

[1] DIN 51309 - Kalibrierung von Drehmomentmessgeräten für statische Drehmomente, Ausgabe 2005-12, Beuth-Verlag GmbH, Berlin
English issue: DIN 51309:2005-12 (E) - Calibration of static torque measuring devices

[2] “Evaluation of measurement data – Guide to the expression of uncertainty in measurement”, JCGM 100:2008

[3] Evaluation of measurement data –Supplement 1 to the “Guide to the expression of uncertainty in measurement” – Propagation of distributions using a Monte Carlo method, JCGM 101:2008

[4] G. Ramm, Kalibrieren von Brückennormalen für die Dehnungsmeßstreifen-Meßtechnik, Messtechnische Briefe 24 (1988), Issue 1, S. 10-16

[5] G. Ramm, Untersuchung des Fehlerverhaltens eines Brückennormals vom Typ BN 100, PTB-Bericht PTB-E-30, Physikalisch-Technische Bundesanstalt; April 1987