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PRACTICAL UNCERTAINTY BUDGETS FOR SPECTRAL MEASUREMENTS OF LEDS

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ABSTRACT

LEDs are used in many applications, ranging from signalling to ambient lighting to displays. Specifications and tolerances are used in the selection of LEDs and these, in turn, are achieved by actual measurements. Uncertainties associated with measurements of LEDs are extremely important, since they define practical limits on specification and quality control expectations. They can also establish whether differences in values between measurements are the result of expected variations or discrepancies in measurement technique, ensuring the litigation is kept to a minimum.

The essentials of uncertainty calculation can be found in the Guide to the Expression of Uncertainty in Measurement *(GUM)*.[1] Unfortunately, most people have not read this document and some that have found it incomprehensible. Several conferences and symposia have addressed the measurement and calculation of uncertainty, but confusion persists. Some confusion stems from the mathematical content of the subject, but most comes when translating a general document, e.g. the GUM, into a specific set of tests aimed at providing values for a specific measurement. Common question that arise are:

- What components of uncertainty are significant?
- How are they determined?
- How can the result be verified?

To some extent experience is essential to answer these questions. However, by choosing specific quantities, device under test and equipment we can illustrate the process by example. The quantities are LED measurements of CIE Condition B averaged LED luminous intensity and luminous flux. The device under test is a temperature regulated LED and the equipment used is a commercial CCD spectroradiometer. Using a spectral irradiance standard in conjunction with limiting apertures of precisely known area allows calibration and measurement of both parameters. Spectroradiometer measurements are traceable to the NIST spectral irradiance scale. This particular LED was chosen because it also has an independent NIST traceable calibration via another path and can provide verification of measurements.

Keywords: LED, spectral, radiant, luminous, intensity, flux, uncertainty, budget, verification, example.

INTRODUCTION

All measurement results have associated uncertainties. This means that the result is not an exact number, but instead a range of values within which the "true" value is asserted to lie.[2] When providing, exchanging or comparing results it is becoming more frequently necessary to determine measurement uncertainties. This is new to many people, and is often presented in mathematical language.[3] There is an understandable confusion *(almost an uncertainty)* associated with the process of preparing uncertainty budgets.

A prime requirement in preparing an uncertainty budget is to be realistic. The best way of doing this is to actually make several measurements and use their variations to calculate uncertainties. These are type A uncertainties. Everything else are type B uncertainties, and include estimates based on similar measurements, experience or manufacturer's specifications.

Much of the trepidation that exists in providing an uncertainty budgets arises from this estimation part of the process. "How do I know what is right?" is a common question. The answer is: if reasonable estimates are not known for any component of uncertainty, type A measurements should be performed where possible. Verification of the total uncertainty budget can be achieved by measuring the same quantity with a different technique or procedure, preferably making use of different types of NIST traceable standards.

DESCRIPTION	*U*	UNIT	TYPE
NIST Irradiance Std	0.42	%	B

DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
	inf	1	%/%	0.42%

Table 1. Headings and one component contribution of a simple uncertainty budget.

To introduce uncertainty budgets, Table 1 shows a simple uncertainty budget layout. Columns should include:

- ***Description*** - A description of each uncertainty component
- ***u*** - The standard uncertainty associated with the component
- ***unit*** - The unit of the uncertainty *(e.g. wavelength may show "nm")*
- ***Type*** - A or B
- ***Distribution*** - If the type is B, how are values distributed. e.g.
  Rectangular - *All values equally likely within limits*
  Normal - *Values are more likely to be around the mid-point*
- ***DOF*** - Degrees of freedom.
  The number of scans-1 for type A, infinite for type B,
- ***Sensitivity*** - How fast the result changes with changes in this component.
- ***Contribution*** - u * Sensitivity

The idea that uncertainty is a property of the measurement equipment is a common misconception. Procedure is an integral part of uncertainty. For instance, a procedure that gives the average of several measurements would be expected to give lower uncertainties than a similar procedure involving a single measurement. In the following examples, a temperature-regulated LED is measured to give values of CIE Condition B averaged LED intensity and flux.[4]

CIE CONDITION B AVERAGED LED SPECTRAL RADIANT INTENSITY

Using a high quality array spectroradiometer, measurements of CIE condition B averaged LED spectral radiant intensity were made with respect to a NIST traceable standard of spectral irradiance. A baffle tube specifically designed for this measurement, which incorporated an integrating sphere with a 1 cm² circular aperture was used. The following procedure was followed:

a) The standard lamp was aligned at the correct distance from the 1 cm² aperture, so the spectral irradiance at the aperture was accurately known. The precise required current was applied to the lamp and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

b) The standard lamp was removed and the LED aligned to a distance of 10 cm from the 1 cm² aperture. The precise required current was applied to the LED and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

c) The LED was rotated in its holder by 90˚. Measurements of signal vs. wavelength were made 20 times.

d) Step c. was repeated for angles of 180˚ and 270˚.

e) The LED was removed and a verification lamp aligned in its place. This verification lamp does not require known values or be at a precise location. All that is required is that it is stable and reproducible each time it is aligned. An arbitrary but precise and consistent current was applied to the verification lamp and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

The spectral irradiance response of the system, calculated from step a, is related to the response to condition B averaged LED spectral radiant intensity by a simple geometric factor. Scans performed in following steps may be converted directly to results of condition B averaged LED spectral radiant intensity. Uncertainty budgets should take into consideration:

- The variation in results, e.g. scan to scan repeatability, realignments, drifts in samples or the measurement system.
- Conformance of the procedure and equipment to the specifications and requirements of condition B averaged LED intensity
- Effects due to the differences *(size, shape, signals, etc.)* between the standard lamp and LED.
- Environmental effects, e.g. changes in temperature and humidity, stray light due to reflections etc.
- Accuracy and stability of operating conditions, e.g. current supplies

Some uncertainties can be minimized by tight control of changes, e.g. temperature. Others can be minimized by optimisations prior to measurements, e.g. minimizing stray light reflections from walls etc. Systematic effects, e.g. signal linearity, stray light within the spectroradiometer, etc. can be corrected and if done only the residual uncertainty after correction need be included. Table 2 gives the uncertainty budget for a result at 523 nm *(close to the peak wavelength)* for the test LED.

DESCRIPTION	*U*	UNIT	TYPE	DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
NIST Irradiance Std	0.42	%	B		inf	1	%/%	0.42%
Transfer to Working Std	0.30	%	B		inf	1	%/%	0.30%
Scan Repeatability	0.05	%	A		19	1	%/%	0.05%
Mechanical Axis	0.13	%	A		3	1	%/%	0.13%
Procedure Reproducibility	0.20	%	A		4	1	%/%	0.20%
LED Distance	0.14	%	B	Rectrangular	inf	2	%/%	0.14%
Aperture Area	0.64	%	B	Rectrangular	inf	0.024	%/%	0.64%
Response Uniformity	0.10	%	A		58	1	%/%	0.10%
Std Lamp Distance	0.07	%	B	Rectrangular	inf	2	%/%	0.07%
LED Current Regulation	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
Std Lamp Current	0.01	%	B	Rectrangular	inf	4	%/%	0.01%
Wavelength Accuracy	0.07	nm	A		6	0.94	%/nm	0.07%
Signal Linearity	0.05	%	B	Normal	inf	1	%/%	0.05%
Stray Light	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
							Combined Uncertainty [in Quadrature] =	**0.66%**
							Expanded Uncertainty at k=2	**1.32%**

Table 2. Uncertainty budget for CIE condition B averaged LED spectral radiant intensity at 523 nm using the procedure above repeated 5 times.

This is just for one wavelength. There is an uncertainty budget for each wavelength, but the preparation of *(and reading of)* potentially hundreds or thousands of budgets to cover spectral measurements is too unwieldy. Having given an example budget to show what is included, it is better to show total uncertainties of spectral values in graphical form.

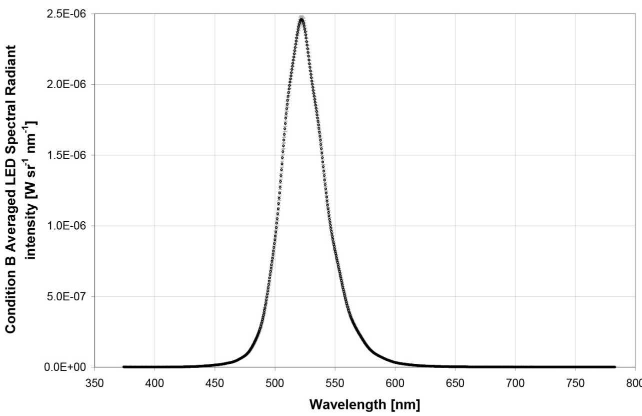

Figure 1. Graphical representation of spectral results. Error bars indicate total expanded uncertainty at k=2.

All the contributions in Table 2 are combined in quadrature, which means the square root of the sum of squares. The expanded uncertainty is expressed as a relative number [%], so to plot it conversion to absolute uncertainties is required. However, as spectral values approach zero the uncertainties do not approach zero. This means the uncertainty budget must include both relative and absolute uncertainty components. The rule for combination of uncertainties is: for quantities that are added or subtracted, combine absolute uncertainties; for quantities that are multiplied or divided, combine relative uncertainties. For a measurement equation such as:

$R = \dfrac{A-d}{B-d}$, which is typical of the ratio of signals with dark subtraction, the numerator and denominator uncertainties are first calculated by combining absolute uncertainties then the ratio uncertainty is calculated from relative numerator and denominator uncertainties. If Figure 1 is zoomed close to zero value, the necessity of expressing final uncertainties as absolute numbers is seen as the uncertainty exceeds the measured value.

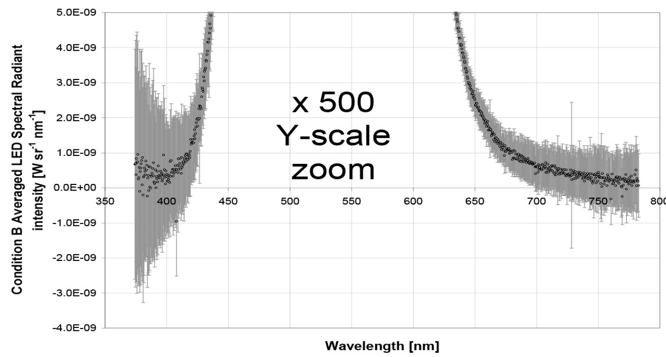

Figure 2. As Figure 1, but with the y-axis zoomed by 500 times to show values and error bars around zero

VERIFICATIONS AND CORROBORATIONS

Step e of the procedure was the measurement of a highly stable and reproducible lamp: a verification lamp. In addition, the LED being measured had previously been calibrated by an independent laboratory for CIE condition B averaged LED intensity using a high accuracy photometer against a NIST calibrated LED of the same type. The NIST calibrated LED also provided a different traceability path to NIST by using a similar procedure *(a - e above)* but the irradiance standard for shape only and the LED standard for absolute scaling. When setting up a measurement procedure for critical results, verifications and collaborations such as these three paths provide are desirable.

CONDITION B [CD]		ROTATION OF LED				MEAN	% U	AVERAGE % U	VERTIFICATION LAMP
		0	90	180	270				
Entire Procedure	Cycle 1	5.3860	5.3290	5.3598	5.3276	5.3506	0.26%	0.31%	0.6703
	Cycle 2	5.3909	5.3353	5.3774	5.3660	5.3674	0.22%		0.6701
	Cycle 3	5.4228	5.3299	5.3757	5.3744	5.3757	0.35%		0.6697
	Cycle 4	5.4376	5.3515	5.4186	5.4251	5.4082	0.36%		0.6692
	Cycle 5	5.4074	5.3078	5.3483	5.3588	5.3556	0.38%		0.6700
Mean		5.4089	5.3307	5.3760	5.3704	5.3715			0.6699
% u		0.18%	0.13%	0.22%	0.29%				0.03%
Average % u					0.21%				

Table 3. CIE condition B averaged LED luminous intensity results from the above procedure in section 2.0 repeated 5 times.

From the verification lamp results, excellent procedure cycle reproducibility was obtained. This is the primary purpose of the verification lamp: to indicate the stability of the procedure results. In fact, two completely different input optic systems *(but of the same type)* were used during these tests without any obvious shift in results. Results from the LED showed a slight drift in results between cycles, possibly indicating small gradual changes of the LED. The rotation of the LED indicates variations due to mechanical axis alignment and cycle variations indicate procedure reproducibility.

DESCRIPTION	*U*	UNIT	TYPE	DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
NIST Irradiance Std	0.39	%	B		inf	1	%/%	0.39%
Transfer to Working Std	0.30	%	B		inf	1	%/%	0.30%
Scan Repeatability	0.0042	%	A		19	1	%/%	0.0042%
Mechanical Axis	0.31	%	A		3	1	%/%	0.31%
Procedure Reproducibility	0.21	%	A		4	1	%/%	0.21%
LED Distance	0.14	%	B	Rectrangular	inf	2	%/%	0.28%
Aperture Area	0.64	%	B	Rectrangular	inf	0.024	%/%	0.02%
Response Uniformity	0.10	%	A		58	1	%/%	0.10%
Std Lamp Distance	0.07	%	B	Rectrangular	inf	2	%/%	0.14%
LED Current Regulation	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
Std Lamp Current	0.01	%	B	Rectrangular	inf	4	%/%	0.04%
Wavelength Accuracy	0.07	nm	A		6	1.7	%/nm	0.12%
Signal Linearity	0.05	%	B	Normal	inf	1	%/%	0.05%
Stray Light	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
							Combined Standard Uncertainty =	**0.72%**
							Expanded Uncertainty at k=2	**1.43%**

Table 4. Uncertainty budget for CIE condition B averaged LED luminous intensity using the procedure above repeated 5 times.

Table 4 shows the uncertainty budget for the procedure *(a - e)* above. Another common method of calibration is to use broadband irradiance standards to provide "shape" and NIST traceable calibrated LEDs to provide absolute scaling. By altering the procedure to incorporate this extra step *(essentially the same as b but with the standard LED in place)* we change the uncertainty budget quite dramatically. This new budget is shown in Table 5. What has changed is not the components but the sensitivities. For instance, in Table 4 the calibration is sensitive, by virtue of the inverse- square law, to the distance of the irradiance calibration lamp. In Table 5, the sensitivity is zero because the spectral shape is essentially constant with changes in distance. Components relating to LEDs increase by the square root of 2 because 2 sets of measurements are now required, except for the distance *(this is almost zero if the tips of the LEDs are at the same distance)*.

DESCRIPTION	*U*	UNIT	TYPE	DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
NIST LED	1.11	%	B		inf	1	%/%	1.11%
NIST Irradiance Std	0.39	%	B		inf	1	%/%	0.39%
Transfer to Working Std	0.30	%	B		inf	1	%/%	0.30%
Scan Repeatability	0.0074	%	A		19	1	%/%	0.0074%
Mechanical Axis	0.31	%	A		3	1.41	%/%	0.45%
Procedure Reproducibility	0.21	%	A		4	1	%/%	0.21%
LED Distance	0.14	%	B	Rectrangular	inf	0.01	%/%	0.00%
Aperture Area	0.64	%	B	Rectrangular	inf	0.024	%/%	0.02%
Response Uniformity	0.10	%	A		58	1.41	%/%	0.14%
Std Lamp Distance	0.07	%	B	Rectrangular	inf	0	%/%	0.00%
LED Current Regulation	0.01	%	B	Rectrangular	inf	1.41	%/%	0.01%
Std Lamp Current	0.01	%	B	Rectrangular	inf	4	%/%	0.04%
Wavelength Accuracy	0.07	nm	A		6	1.7	%/nm	0.12%
Signal Linearity	0.05	%	B	Normal	inf	1	%/%	0.05%
Stray Light	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
							Combined Standard Uncertainty =	**1.32%**
							Expanded Uncertainty at k=2	**2.64%**

Table 5. Uncertainty budget for CIE condition B averaged LED luminous intensity using the modified procedure, incorporating a NIST traceable calibrated LED, repeated 5 times.

The NIST LED adds a dominant contribution to the uncertainty budget, and almost doubles the total uncertainty despite the reduced contributions from other components.

Figure 3 shows the final values, with error bars to indicate k=2 uncertainties, from the two methods plus values from an independent laboratory. The close agreement provides corroboration of values. The differences in uncertainty indicate the procedure *(a - e)* above involving just the irradiance standard is the most accurate of the 3 methods.

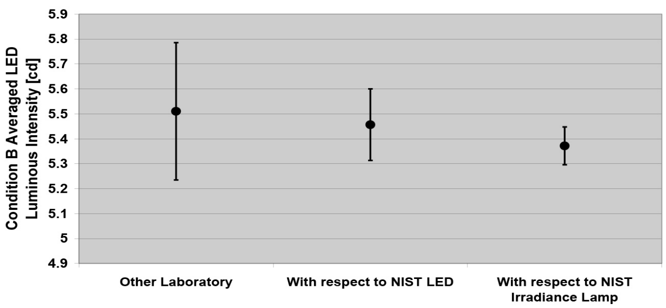

Figure 3. Results with uncertainties at k=2 (shown as error bars) for 3 methods of determining condition B averaged LED luminous intensity for the same LED. The Other Laboratory used a photometer method.

LED SPECTRAL FLUX AND LUMINOUS FLUX

Measurements of LED total flux should be made at the center of a large integrating sphere or using a goniometer. Measurement of forward-looking flux (also called 2π flux), where the LED is at the side wall of an integrating sphere is also common. The temperature controlled LED housing blocks backward emission, so for this LED the total flux and forward-looking flux are the same. The following procedure therefore measured the LED at the side wall of a specially designed integrating sphere for ease.

a) The standard lamp was aligned at the correct distance from a precision aperture, so the spectral irradiance at the aperture was accurately known. Light that passed through the aperture entered the sphere entrance port. The precise required current was applied to the lamp and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

b) The standard lamp was turned off. An auxiliary lamp, permanently housed in the sphere, was switched on and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

c) The standard lamp and precision aperture were removed and the LED was placed at the sphere entrance port. The auxiliary lamp was still on, and measurements of signal vs. wavelength were made 20 times.

d) The auxiliary lamp was switched off. The precise required current was applied to the LED and, following a suitable warm-up period, measurements of signal vs. wavelength were made 20 times.

e) The LED was rotated in its holder by 90˚. Measurements of signal vs. wavelength were made 20 times.

f) Step c. was repeated for angles of 180˚ and 270˚.

The auxiliary lamp, used in steps b and c, is to compensate for interactions between the sphere and LED. Integrating spheres work because light within them reflects from the walls many times. Anything placed in them or against their ports can absorb or reflect light and because of the sphere's multiple reflections, produce an effect out of proportion to its size. It is therefore important to compensate for the interaction in order to produce accurate results.

The previous discussions on correct expression of CIE averaged LED spectral radiant intensity results also apply to LED spectral flux measurements.

VERIFICATIONS AND CORROBORATIONS

Using a similar approach to the CIE condition B averaged LED luminous intensity; we can construct uncertainty budgets and use different types of NIST traceable calibration standard in procedures to provide corroboration. Also, the Other Laboratory provided measurements of total luminous flux *(TLF)* for the LED.

CONDITION B [CD]		ROTATION OF LED				MEAN	% U	AVERAGE % U
		0	90	180	270			
Entire Procedure	Cycle 1	0.7941	0.7941	0.7952	0.7953	0.7947	0.043%	0.043%
	Cycle 2	0.7944	0.7943	0.7956	0.7955	0.7950	0.043%	
	Cycle 3	0.7932	0.7933	0.7945	0.7940	0.7938	0.039%	
	Cycle 4	0.7893	0.7892	0.7903	0.7908	0.7899	0.051%	
	Cycle 5	0.7873	0.7873	0.7885	0.7889	0.7880	0.051%	
	Cycle 6	0.7862	0.7861	0.7872	0.7869	0.7866	0.033%	
Mean		0.7908	0.7907	0.7919	0.7919	0.7913		
% u		0.19%	0.19%	0.19%	0.19%			
Average % u					0.19%			

Table 6. TLF results from the above procedure in section 3.0 repeated 6 times.

The results of the procedure, shown in Table 6, indicate that the result was not very dependent on LED rotation and had similar cycle reproducibility to condition B averaged LED intensity measurements. Again, this may reflect small slow variations in the intensity of the LED itself.

DESCRIPTION	*U*	UNIT	TYPE	DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
NIST Irradiance Std	0.39	%	B		inf	1	%/%	0.39%
Transfer to Working Std	0.30	%	B		inf	1	%/%	0.30%
Scan Repeatability	0.0069	%	A		20	1	%/%	0.0069%
Mechanical Axis	0.04	%	A		4	1	%/%	0.04%
Procedure Reproducibility	0.19	%	A		5	1	%/%	0.19%
LED Distance	0.01	%	B	Rectrangular	inf	2	%/%	0.02%
Aperture Area	0.23	%	B	Rectrangular	inf	1	%/%	0.23%
Response Uniformity	0.15	%	A		59	1	%/%	0.15%
Std Lamp Distance	0.07	%	B	Rectrangular	inf	2	%/%	0.14%
LED Current Regulation	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
Std Lamp Current	0.01	%	B	Rectrangular	inf	4	%/%	0.04%
Wavelength Accuracy	0.07	nm	A		7	1.7	%/nm	0.12%
Signal Linearity	0.05	%	B	Normal	inf	1	%/%	0.05%
Stray Light	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
							Combined Standard Uncertainty =	**0.62%**
							Expanded Uncertainty at k=2	**1.25%**

Table 7. Uncertainty budget for TLF using the procedure in section 3.0 repeated 6 times.

Table 7 is very similar to Table 4 despite the fact that different parameters are being measured. This because some of the system components were the same and there are similarities in the procedures. Even so, some of the sensitivities are different. For instance, the aperture area is used in both calibration and measurement of averaged LED intensity but is only used in the calibration step for spectral flux. This means uncertainties in the aperture area have a direct effect on the TLF result and hence the sensitivity is 1. Response uniformity also has a different meaning in Tables 4 and 7. In Table 4 it was the spatial uniformity across the 1 cm^2 aperture, but in Table 7 it is the uniformity of the sphere to light at different angles from the LED position.

DESCRIPTION	*U*	UNIT	TYPE	DISTRIBUTION	DOF	SENSITIVITY		CONTRIBUTION
NIST LED	0.42	%	B		inf	1	%/%	0.42%
NIST Irradiance Std	0.39	%	B		inf	1	%/%	0.39%
Transfer to Working Std	0.30	%	B		inf	1	%/%	0.30%
Scan Repeatability	0.0085	%	A		19	1	%/%	0.0085%
Mechanical Axis	0.04	%	A		3	1.41	%/%	0.06%
Procedure Reproducibility	0.19	%	A		4	1	%/%	0.19%
LED Distance	0.01	%	B	Rectrangular	inf	1.41	%/%	0.01%
Aperture Area	0.23	%	B	Rectrangular	inf	0	%/%	0.00%
Response Uniformity	0.15	%	A		58	1.41	%/%	0.21%
Std Lamp Distance	0.07	%	B	Rectrangular	inf	0	%/%	0.00%
LED Current Regulation	0.01	%	B	Rectrangular	inf	1.41	%/%	0.01%
Std Lamp Current	0.01	%	B	Rectrangular	inf	4	%/%	0.04%
Wavelength Accuracy	0.07	nm	A		6	1.7	%/nm	0.12%
Signal Linearity	0.05	%	B	Normal	inf	1	%/%	0.05%
Stray Light	0.01	%	B	Rectrangular	inf	1	%/%	0.01%
							Combined Standard Uncertainty =	**0.72%**
							Expanded Uncertainty at k=2	**1.44%**

Table 8. Uncertainty budget for TLF using the modified procedure in section 3.0, incorporating a NIST traceable calibrated LED, repeated 6 times.

When the NIST calibrated LED is added to the spectral flux procedure, similar to section 2.1, the uncertainties change to those shown in Table 8. Again, the effect is to change the sensitivities. This time, the additional uncertainty due to the NIST LED is smaller so the two methods have almost the same uncertainties.

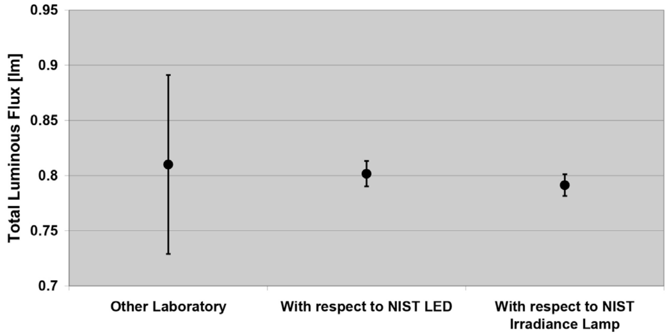

Figure 4. Results with uncertainties at k=2 (shown as error bars) for 3 methods of determining TLF for the same LED. The Other Laboratory used a photometer method.

Figure 4 shows the results, with error bars representing k=2 uncertainties. The Other Laboratory reported reproducibility concerns with their equipment and hence could not achieve equivalent performance to this present study.

CONCLUSIONS

The use of good quality equipment together with appropriate procedures can lead to low uncertainties. More importantly, the uncertainties can be quantified and expressed as an uncertainty budget. Techniques for preparing uncertainty budgets have been discussed. Changes to the procedure can affect the uncertainties. In particular, procedures that follow different traceability paths to NIST standards may provide different uncertainties. In measurements of both CIE condition B averaged LED intensity and total flux, it was found that direct calibration to tungsten halogen lamps traceable to the NIST irradiance scale gave the best results of those methods discussed.

When first preparing uncertainty budgets and trying to understand the concepts it can be quite daunting. Nevertheless, uncertainty budgets and the need to express measurement results as more than a single number are becoming more common and necessary. It is hoped that the examples provided here will help.

[1] International Organization for Standardization: Guide to the Expression of Uncertainties in Measurements, ISO 1995

[2] Royal Society of Chemistry: Terminology – the key to understanding analytical science. Part 1: Accuracy, precision and uncertainty, AMC Technical Brief 13, September 2003, (Available on www.rsc.org/lap/rsccom/amc/amc_index.htm).

[3] Commission Internationale de l'Éclairage: Determination of measurement uncertainties in photometry, (currently in preparation by CIE TC2-43)

[4] Commission Internationale de l'Éclairage: Measurement of LEDs, CIE 127-1997.